Continuous change

دفتر :

تحليل عددي

Numerical Analysis

الاسبوع 5-1

للطالبة لين حيدر

اللجنة الأكاديمية لقسم الهندسة الصناعية

2025

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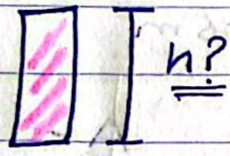
Errors :-



→ 130 cm (1^{st})
150 cm (2^{nd})
152 cm (3^{rd})

aproximate values

Types of errors :-

I) True value (E_T)

(E_T) = True value − Aprox. value

II) Aproximate value (E_a)

(E_a) = New value − old value

III) Relative True error (ϵ_T)

$$(\epsilon_T) = \left| \frac{(E_T)}{\text{True value}} \right| \times 100\%$$

IV) Relative Aproxi. error (ϵ_a)

$$(\epsilon_a) = \left| \frac{(E_a)}{\text{New value}} \right| \times 100\%$$

Note

Type (one & two) could be (− or +) but (3,4) could'nt, only +

Ex: find the E_T, ϵ_T for the following areas

True	5m
10m	

$A_T = 50m^2$

Aprox	5.06 m
10.2m	

$A_a = 51.6 m^2$

$(E_T) = 50 - 51.6 = (-1.6)$

$(\epsilon_T) = \left| \frac{(E_T)}{\text{True value}} \right| \times 100\%$

$= \left| \frac{1.6}{50.0} \right| \times 100\% = (3.2\%)$

Ex: we used a nu. method to aproximate the (True) value root of:

$f(x) = x^2 - 4x - 5$

i	x_r
1	3.9
2	4.6
3	4.83
4	4.94

→ Find E_T, ϵ_T, E_a, ϵ_a After the (4th) itteration :-

(1) (E_T) = True.v − aprox.v

(root) then ⟹ $5 - 4.94 = (.06)$

$x^2 - 4x - 5 = 0$

$(x-5)\ (x+1) = 0$

$x = 5$ accepted

$(x = -1)$ rejected

(2) $(\epsilon_T) = \left| \frac{E_T}{\text{True.v}} \right| \times 100\%$

$= \left| \frac{.06}{5} \right| \times 100\% = 1.2\%$

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3) Aproxi.valu $\Rightarrow$ (Ea) = New (4th) − old.v (3rd)

4.94 − 4.83 = .11

4) (Ea) = $\left| \frac{Ea}{New.v} \right| \times 100\% \Rightarrow \frac{.11}{4.94} \times 100\% = 2.27\%$

Taylor series:-

[fun تعريف معروف ولا واحد ... بين قيمة Apraxi ل (real) بين] $\rightarrow f(x)$ [$f(2) = 4$, $2 \rightarrow x$]

مشتقة

$$f(x) = \sum_{i=0}^{n} \frac{(x - x_0)^i f^i(x)}{i!}$$

[$f(4) = ?$, $4 \rightarrow x$]

$(i=0)$ $\frac{(x - x_0)^0 f^0(x)}{0!}$ المشتقة صفر يعني الاقتران نفسه $\Rightarrow f(x)$

const!! (zero order equ)

$(i=1)$ $\frac{(x - x_0)^1 f^1(x)}{1!} = (x - x_0)^1 f^1(x)$

(first order equ)

$(i=2)$ $\frac{(x - x_0)^2 f^2(x_0)}{2!} = \frac{(x - x_0)^2 f^2(x_0)}{2}$

(second order equ)

$$f(x) = \sum \Rightarrow f^0(x) + (x - x_0)^1 f^1(x_0) + \frac{(x - x_0)^2 f^2(x)}{2} \cdot$$

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Ex:- $f(x)=\sin(x)$ in a taylor series & use it to aproximate the value of $\sin(\overset{x}{2})$ using the value of $f(x)$ & it's derivatives at $\underline{\underline{00}}\ x_0$ using 4 signficanel figur the value of $f(x)$ & it's first three derivative are?

(X_0) المطلوب، (X) ← المعطى بالسؤال

$f(00)=?$ $\sin(2)$

$f^0(x) = \sin(x) \Rightarrow \sin(0) \rightarrow \underline{zero}$

$f^1(x) = \cos(x) \Rightarrow \cos(00) = \underline{\underline{1}}$

$f^2(x) = -\sin(x) \Rightarrow -\sin(00) = \underline{zero}$

$f^3(x) = -\cos(x) \Rightarrow -\cos(00) = \underline{\underline{-1}}$

$\boxed{2!}$ $\boxed{00}$ $\boxed{-\frac{1}{3}}$

$f(x) = zero + \frac{(2-0)^{2} \times \boxed{1}}{1!} + 2 \times zero + \cancel{\frac{-2}{6}}$

$2 - \frac{1}{3} = \underline{\underline{1.666}}$

(E_T) = True.v − ApXi.v

$.0348 - 1.666 = [1.6312]$

Ex $f(x) = \sin(x) + x^2$ given $f(\frac{\pi}{3}) = ?$

$x_0 = \frac{\pi}{4}$

$f^0(x_0) = \sin(\frac{\pi}{4}) + (\frac{\pi}{4})^2 = \underline{1.324}$

$f^1(x_0) = \cos(x_0) + 2x \Rightarrow \cos(\frac{\pi}{4}) + 2\frac{\pi}{4 \times 2} = \underline{2.985}$

$f^2(x_0) = -\sin(x_0) + 2 \Rightarrow -\sqrt{2} + 2 = \underline{1.2928}$

$f^3(x_0) = -\cos(x_0) + 0 \Rightarrow -\sqrt{2} = \underline{-0.707}$

$f^4(x_0) = \sin(x_0) \rightarrow \sqrt{2} = 0.707$

3!

$\hookrightarrow f(x) = 1.324 + \left(\frac{\pi}{3} - \frac{\pi}{4}\right) \cdot 2.278 + \left(\frac{\pi}{3} - \frac{\pi}{4}\right)^2 \cdot \frac{1.293}{2!}$
$+ \ominus \left(\frac{\pi}{3} - \frac{\pi}{4}\right)^3 \cdot \frac{.707}{6} \Rightarrow \boxed{1.9631}$

Ex $f(x) = x^4 - 3x^2 + 1$ (given) $x_0 = 1$
$x = 2$

$f^0(x_0) = -1$ / $f^1(x) = 4x^3 - 6x$
$f^1(x_0) = \boxed{-2}$

$f^2(x_0) = 12x^2 - 6$ / $f^3(x_0) = 24x$ $\boxed{24}$
$\boxed{6}$

$\hookrightarrow [f^4(x_0) = 24] \leftarrow$ exactly !!

$f(x) = -1 + \frac{-2}{1} + \frac{\overset{3}{\cancel{6}} \cdot 1}{\cancel{2}\,2!} + \frac{\overset{4}{\cancel{24}} \cdot 1}{\cancel{3!}\,4} = \boxed{4}$

$\hookrightarrow [f^5(x_0) = 00] \leftarrow$
$\hookrightarrow$ final turm !!

$\hookrightarrow$ Root of equ

Backeting:
- $\longrightarrow$ Bisection Method
- $\longrightarrow$ False position Method

open:
- $\longrightarrow$ simple Fixed point iteration
- $\longrightarrow$ Newton Raphson Method
- $\longrightarrow$ Secant Method..

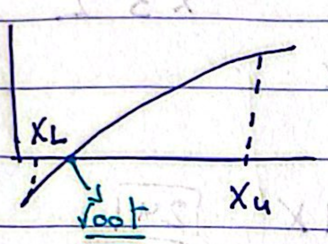
$\hookrightarrow$ The x's values that Make the equ = zero or the graph to cross the x-axis

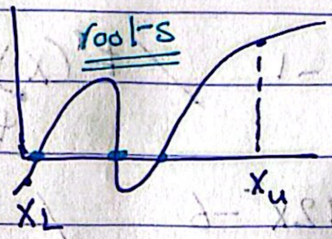
BRACKETING Method

rule a: $(X_L \;\&\; X_u)$ having two roots

rule b: $f(X_L) \cdot f(X_u) < 0$

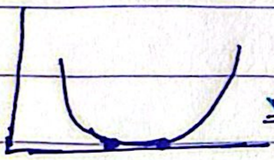
rule c: Continous function ✓



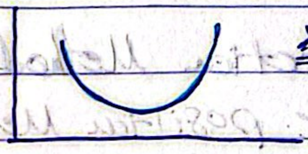


$f(X_L) \cdot f(X_u) > 0$

حيث لو فين وجدت الية
رح يطلع x وهذا لان
الاقتران لم يقطع (axis-X)



✓



✗ : Can't be solve in this method!

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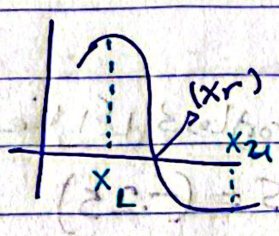
تمرين

Bisection Method :-

(a) check on $f(X_L) \cdot f(X_u) < 0$

(b) $\left[X_{r_1} = \frac{X_L + X_u}{2} \right] \rightarrow$ حفظ

لو قال لك بالنسبة للـ accepted error



لو (Xr) بتحل مكان اللي يكون نفس الاشارة معه
(- ← -)
(+ ← +)

Continuous ✓

$f(x_L)$ + / $f(x_u)$ -

$$\left[X_{r_1} = \frac{X_L + X_u}{2} \right]$$

(E_s) يعني نسبة الخطأ المسموح بها (E_s)

Ex find an estimate for ($\sqrt[4]{25}$) with in (.05) from the root starting with $[2, 3]$ (x_L, x_u) using by Light placed.

($\sqrt[4]{25}$) → function $\underline{x}$ ما فيه

$\sqrt[4]{25} = \underline{x}$ ← لذلك نعمل

$x^4 = 25 \rightarrow (x^4 - 25 = 0)$: لو عندي function فيها $\underline{x}$

$\rightarrow x^4 - 25 = 0 \rightarrow f(2) = -9$ ✓

$f(3) = 56$ ✓

Rule ✓

$x_{r_1} = \frac{2+3}{2} = (2.5)$ $f(2.5) = +14.66$

لـ راح تحل مكانه ال (X_u)

لانو موجب (+)

$(X_L = 2)$ $(X_u = 2.5)$

$\hookrightarrow X_{r_2} = \frac{2.5+2}{2} = 2.25$

لـ نضل نحل لحد ما (error) يوصل لل eccepted او اقل :

$\hookrightarrow E_a = New - old \Rightarrow 2.25 - 2.5 = (-.25)$

② Bisection Method

i	X_L	X_u	$f(X_L)$	$f(X_u)$	X_r	$f(X_r)$	error
1	2	3	-9	-9	2.5	14.06	—
2	2	2.5	-9	14.06	2.25	.65	.25
3	2	2.25	-9	.63	2.125	-4.6	0.125
4	2.125	2.25	-4.6	,63	2.1875	-2.1022	.0625
5	2.1875	2.25	-2.1022	,63	$\boxed{2.21875}$	-.7655	.0312

(answer) بوقف

$$E_s = \frac{X_u - X_L}{2^n}$$ n → number of iteration

لـ قانون عشان اعرف كم iteration بدي

$$\underline{n} = \frac{\ln(X_u - X_L / E_s)}{\ln 2}$$

(given)

لـ بيقرب للاكبر دايماً

لـ قانونين عشان الرقم ميكون (error) , (Es)

فش (Es) لو كانت (Es) .

تفتش لما صورتين $(\mp \mp)$
او ماضي فيه
او مش محصورة

false-position Method

الوحيد بعد Bisction upper / lower

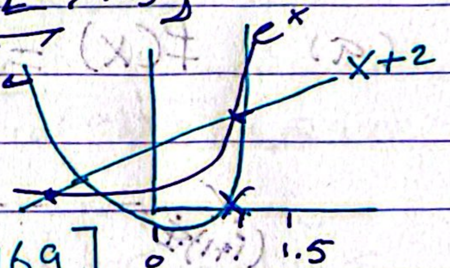
$$\left[X_r = X_u - \frac{f(x_u)\,(x_u - x_L)}{f(x_u) - f(x_L)} \right] \text{ (given)!}$$

ممكن نطرح (x_L)

تقاطع $\ominus$

Ex :- find the intersection point between the two function $f(x) = e^x$ & $f_2(x) = x+2$ $E_s < .02$ starting with $[0, 1.5]$

نقاط الاقتران
تقاطع



$f(x) \Rightarrow \boxed{e^x = x+2}$

$[f(0) = -1 \quad / \quad f(1.5) = 0.98169]$

i	X_L	X_u	$f(X_L)$	$f(X_u)$	X_r	$f(X_r)$	Error
1	0	1.5	-1	.98169	.75693	-0.6252	—
2	.75693	1.5	-.6252	.98169	1.04604	-.19968	.2911
3	1.6460	1.5	-0.1997	.98169	1.1227	-0.0495	0.076

ضع (Liner) ← Bisction اسرع

ضع (curve function) ← false اسرع

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↳ Open Methods:-

(1) fixed -point itteration : (ماهي قانون)

given → $f(X) = \sim$

→ $\underline{\underline{X}}$ ↲

$\underline{\underline{x_l}}$ بدل $\left(\begin{matrix} x_l \\ x_u \end{matrix}\right)$ (initial guess)

(a) $f(X) = (0)$ (نحط X وحدة موضوع للقانون)

$X_{(i+1)} = \text{----} \; x_i \rightarrow g(x)$

$(i=0) \Rightarrow (x_1 = \text{--} x_0)$

$(i=1) \Rightarrow (x_2 = \text{----} x_1)$

$\underline{\underline{Ex}}$ $\sin(x) - x = 0$

↳ ~~sin~~ $[x_{(i+1)} = \cancel{\sin}\ \sin(x_i)]$

لما يكون عندي اقتران صعب اطلع (X) موضوع للقانون نضيف (X) للطرفين.

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Ex

$(\sin(x) - x^2)$ $\quad x_0 = 1 \quad \epsilon_s =$ (Relative)

$\sin(x) = x^2 = 0$

$x_{i+1} = \sqrt{\sin(x)_i}$

$(i=0) \longrightarrow x_1 = \sqrt{\sin(x_0)} = \sqrt{\sin(1)} = .9173$

$\epsilon_a = \left| \frac{.9173 - 1}{.9173} \right| * 100\% = 9.015\%$

$(i=1) \longrightarrow x_2 = \sqrt{\sin(x_1)} = \sqrt{\sin(.9173)} = .89104$

لـ كيف اعرف هاي الصورة صالحة استخدمها.

$g(x) = \ldots$

$\longrightarrow \left|\frac{dy}{dx}\right| < 1$ will converge. $\longrightarrow$ Error تقل

$\longrightarrow \left|\frac{dy}{dx}\right| > 1$ will Diverge. $\longrightarrow$ Error بتزيد

$\longrightarrow \left|\frac{dy}{dx}\right| \lessapprox 1$ will converge but slowly.

لـ متى بتفشل الطريقة ؟

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Ex $f(x) = -x^2 + 1.8x + 2.5$

$x_0 = 5 \quad \epsilon_a = .05\%$

(1) $-x^2 + 1.8x + 2.5 = 0$

a $x_{(i+1)} = \sqrt{1.8x_i + 2.5}$ b $x_{i+1} = \frac{-2.5}{1.8 - x_i}$

c $x_{i+1} = \frac{x_i^2 - 2.5}{1.8}$

(2) ($x=5$) . بشتق الصورة عشان اختار الصورة

c $\frac{x^2 - 2.5}{1.8} \Rightarrow \frac{2x_i}{1.8} = 5.55 > 1$ dive

a $\frac{dy}{dx} = \frac{1.8}{2\sqrt{1.8x + 2.5}} = .2653 < 1$ Converge

سؤال

→ best form from them ?

لا لازم اشتق على كل الصور.

لو كل ما كانت الرقم اقل بيكون احسن ويعوض اقل itteration

(i=0) $x_1 = \sqrt{1.8x_0 + 2.5} \Rightarrow 3.3911$

$\epsilon_a = \left| \frac{3.3911 - 5}{3.3911} \right| \times 100\% = 47.44\%$

(i=1) $x_2 = \sqrt{1.8x_1 + 2.5} = 2.933$

$\epsilon_a = \left| \frac{2.933 - 3.911}{2.933} \right| \times 100\% = 15.5\%$

وتكملة

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(2) Newton-Raphson Method. $\rightarrow [f(x), x^\circ]$ given

$$\left[x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \right] \rightarrow$$ مش هاقدر استخدمها إلا (ع.م)

موجود بالمشتقة

$f(x) = x^2 - 1 \longrightarrow x_{i+1} = x_i - \dfrac{x_i^2 - 1}{2x_i}$

Ex $f(x) = \sin(x) - x^2$ $(x_\circ = 1)$ $(\epsilon_s = .1\%)$

(i=0) $x_1 = 1 - \dfrac{\sin(1) - 1}{\cos(1) - 2} = .8913$

$\epsilon_a = \left| \dfrac{.8913 - 1}{.8915} \right| * 100\% = (12\%)$

(i=1) $x_2 = .8913 - \dfrac{\sin(.8913) - (.8913)^2}{\cos(.8913) - 2(.8913)}$

$= .8769$

$\epsilon_a = \left| \dfrac{.8769 - .8913}{.8769} \right| * 100\% = 1.6\%$

(i=2)

$x_3 = .8915 - \dfrac{\sin(.8769) - (.8769)^2}{\cos(.8769) - 2(.8769)}$

$[x_3 = .8768]$

$(\epsilon_a = .011\%) < .1\%$ (Stop)

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(3) Secant Method

$f(x)$, ~~x_0~~ $[x_0]$, x_{-1} (given) (القيمة اللي قبلها معطاة)

given $\left[x_{i+1} = x_i - \frac{f(x_i) \cdot (x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)} \right]$

Ex $f(x) = \sin(x) - x^2$, $x_{-1} = 1$ $\epsilon_a = .1\%$

$x_0 = 2$

$(i=0)$ $x_1 = 2 - \frac{(\sin(2) - 4)(1-2)}{(\sin(1) - 1) - (\sin(2) - 4)} = \boxed{.946}$

$\epsilon_a = \left| \frac{.946 - 2}{.946} \right| * 100\% = 111\%$

$(i=1)$

$x_2 = .946 - \frac{(\sin(.946) - (.946)^2) - (2 - .946)}{\sin(2) - (\sin(.946) - (.946)^2)}$

$= \boxed{.9166}$

$\epsilon_a = \left| \frac{.9166 - .946}{.9166} \right| * 100\% = \boxed{3.2\%}$

$(i=2)$

$x_3 = .9166 - \frac{-.0466(.0946 - 0.9166)}{-0.0831 + .0466}$

كمل ... $\boxed{= .8798}$

/ /

solve → Find the roots (x_r, y_r)

System of non-liner equ:-

$10x^3 + 3y^3 = 23$ | $12x^2 + y^3 = 5$

→ Newton – Raphson method:-

↳ x, y equ ↳ system ↳ $x_\circ, y_\circ$

given: $f_1(x,y)$, $f_2(x,y)$

a) $J = \begin{vmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{vmatrix}$ given

b) $X = \begin{vmatrix} f_1 & \frac{df_1}{dy} \\ f_2 & \frac{df_2}{dy} \end{vmatrix}$

c) $y = \begin{vmatrix} \frac{df_1}{dx} & f_1 \\ \frac{df_2}{dx} & f_2 \end{vmatrix}$ = لازم احسب error لها

d) $x_{i+1} = x_i - \frac{|x_i|}{|J_i|}$ → يكون معوض فيه

e) $y_{i+1} = y_i - \frac{|y_i|}{|J_i|}$

ex use N.R.M to determind roots:-
(x_r, y_r) for :-

$$x^2 + xy = 10 \qquad x_0 = 1$$
$$y + 3xy^2 = 57 \qquad y_0 = 2$$

$$f_1 = x^2 + xy - 10 = 0$$
$$f_2 = y + 3xy^2 - 57 = 0$$

(x_0, y_0)

$$J = \begin{vmatrix} 2x+y & x \\ 3y^2 & 1+6xy \end{vmatrix} \longrightarrow \begin{vmatrix} 4 & 1 \\ 12 & 13 \end{vmatrix}$$

$$X = \begin{vmatrix} x^2+xy-10 & x \\ y+3xy^2-57 & 1+6xy \end{vmatrix} \longrightarrow \begin{vmatrix} 7 & 1 \\ -43 & 13 \end{vmatrix}$$

$$Y = \begin{vmatrix} 2x+y & x^2+xy-10 \\ 3y^2 & y+3xy^2-57 \end{vmatrix} \rightarrow \begin{vmatrix} 4 & 7 \\ 12 & 43 \end{vmatrix}$$

$I=0$

$$x_1 = x_0 - \frac{|X_1|}{|J|} \quad \text{and} \quad \Rightarrow \quad y_1 = y_0 - \frac{88}{40}$$

$$1 - \frac{|-48|}{40} \qquad y_1 = \boxed{4.2}$$

$$x_1 = \boxed{2.2}$$

matrix operations:

	coefficient	variables	result

$$5x + 3y = 2$$
$$4x + 5y = 13$$
$$3x + 4y = 20$$

$$\Rightarrow \begin{bmatrix} 5 & 3 \\ 4 & 5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 13 \\ 20 \end{bmatrix}$$

$a^{1\times1}$ ، $a\ 1\times3$ ، A

في اسم المصفوفة بالكبتل.
$A\ 3\times2$ ← عدد الصفوف x عدد العمود ← size ل
(1×1) $(\underline{a})$ elements ل

⇒ matrix addition. ← لازم يكونو same size

$$\underset{2\times2}{A} = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \qquad \underset{2\times2}{B} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} \qquad C \Rightarrow A \pm B$$

$$\underset{2\times2}{C} = \begin{bmatrix} 3 & 1 \\ 5 & 9 \end{bmatrix}$$

⇒ matrix Multiplication

$$\Rightarrow \underset{3\times\underline{2}}{\underline{A}} \times \underset{\underline{2}\times\_}{\underline{B}} = C$$

عدد الاعمدة في الثانية يساوي عدد صفوف الاول

$A\ 2\times3$ ~~3×2~~ $\qquad B\ 2\times2 \qquad A\times B \neq B\times A$ ل

$$\begin{bmatrix} 1 & 5 \\ 6 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 22 & \\ & \\ & \end{bmatrix}$$

$$C_{3\times2}$$

$$c_{1\times1} = (1\times2) + (5\times4) = \underline{\underline{22}}$$
$$c_{2\times2} = (6\times1) + (2\times3) = 12 \qquad \text{(عامود × صف)}$$
$$c_{3\times2} = (4\times1) + (1\times3) = 7$$

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$\Rightarrow \underset{3\times3}{A} \times \underset{3\times3}{A^{-1}} = \underset{3\times3}{I} \rightarrow$ idintity matrix I

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ matrix transpose $\Rightarrow$ اقلب الصف عامود

$$A_{3\times2} \longrightarrow A^T_{2\times3}$$

$$\underset{3\times2}{A} = \begin{bmatrix} 2 & 1 \\ 0 & 5 \\ 6 & 4 \end{bmatrix} \Rightarrow \underset{2\times3}{A^T} = \begin{bmatrix} 2 & 0 & 6 \\ 1 & 5 & 4 \end{bmatrix}$$

$\Rightarrow$ row operations $\Rightarrow$ a. Row swapping

$$\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix} \Longrightarrow \begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix}$$

بعمل تبديل على الجوانب

b. multiplaction by factor

$$\begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix} \rightarrow \text{row}_1 \times 2 = \begin{bmatrix} 4 & 2 \\ 5 & 6 \end{bmatrix}$$

/ /

c. row addition.

$$A_{2\times 2} = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix} \Rightarrow 2R_1 + R_2 = R_2$$

$$\begin{bmatrix} 2 & 1 \\ 10 & 5 \end{bmatrix}$$

→ gauss elemination

$\underline{\underline{A}}x = \underline{b}$ → coeff. / result

$$\Rightarrow \begin{aligned} 4x_1 + 2x_2 + 5x_3 &= 5 \\ 10x_1 + x_2 + 4x_3 &= 2 \\ 5x_1 + 3x_2 + 5x_3 &= 1 \end{aligned}$$

x_1, x_2, x_3 Liner

a//

$$\begin{bmatrix} 4 & 2 & 3 \\ 10 & 1 & 4 \\ 5 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$$

b// Angmentdd matrix $[A : B]$

$$\left[\begin{array}{ccc|c} 4 & 2 & 3 & 5 \\ 10 & 1 & 4 & 2 \\ 5 & 3 & 5 & 1 \end{array}\right]$$

→ بدي اعمل دا عمليات عشان نصفر بـ p1 (Lower)

(under the diagonal)

→ back_word sub

$0x_1 + 0x_2 + 5x_3 = 1$

(and so on)

/ /

Ex

$$x_1 + x_2 + 6x_3 = 7$$
$$-x_1 + 2x_2 + 9x_3 = 2$$
$$x_1 - 2x_2 + 3x_3 = 10$$

a) matrix → argmented

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 7 \\ -1 & 2 & 9 & 2 \\ 1 & -2 & 3 & 10 \end{array}\right]$$

→ بدي اصفرو ← كيف بدي ← بضرب (factor)

$$\left[-\left(\frac{\text{no. to be zero}}{\text{pivot}}\right) \right]$$

* وكانه يلي فيهم تركيزة
* انا اي صف
* ⟸ وين التصفير

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 7 \\ 0 & 3 & 15 & 9 \\ 0 & 0 & 12 & 12 \end{array}\right]$$

$\hookrightarrow 0x_1 + 0x_2 + 12x_3 = 12$

$\boxed{x_3 = 1}$

$\hookrightarrow 0x_1 + 3x_2 + 15x_3 = 9$

$\boxed{x_2 = -2}$

$\hookrightarrow x_1 + (-2) + 6 = 7$

$\boxed{x_1 = 3}$

لو كان عندي الـ coeff = 0 في Diagonal
هون لازم اعمل (row swapping)

/ /

pivot = 0

row swapping شرط
لازم ابدل row الي فيه اكبر معامل ب X_1
بعض النظر عن 1

$$0X_1 + 2X_2 + 3X_3 = 8$$
$$9X_1 + 6X_2 + 7X_3 = -3$$
$$2X_1 + X_2 + 6X_3 = 5$$

ليش اخذنا اكبر معامل ؟

swap
row swapping

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 7 \\ 0 & 3 & 15 & 9 \\ 1 & -2 & 5 & 10 \end{array}\right]$$

Swap
حلوه zero

Ex

$$2X_2 + 3X_3 = 8 \quad \text{swapping}$$
$$4X_1 + 6X_2 + 7X_3 = -3$$
$$2X_1 + X_2 + 6X_3 = 5$$

$$\left[\begin{array}{ccc|c} 4 & 6 & 7 & -3 \\ 0 & 2 & 3 & 8 \\ 2 & 1 & 6 & 10 \end{array}\right]$$

$$\left(\frac{2}{4}\right) R_1 + R_3 = R_3$$

$$2 \quad 1 \quad 6 \;|\; 5$$
$$0 \quad -2 \quad 2.5 \;|\; 6.5$$

$$\left[\begin{array}{ccc|c} 4 & 6 & 7 & -3 \\ 0 & 2 & 3 & 8 \\ 0 & -2 & 2.5 & 6.5 \end{array}\right]$$

$[2 = -2]$

مهو لازم اشيل

swapping لهون

: كما نعرا على الصف
لو تحاجة swapp ما اعمل zero point

/ /

$$\left[\begin{array}{ccc:c} 4 & 6 & 7 & 3 \\ 0 & 2 & 3 & 8 \\ 0 & 0 & 5.5 & 14.5 \end{array}\right]$$

back word subsutional

$\hookrightarrow 0X_1 + 0X_2 + 5.5X_3 = 14.5$

$\boxed{X_3 = 2.64}$

$\hookrightarrow X_1 + 2X_2 + 3X_3 = 8$

$\boxed{X_2 = .04}$

$\hookrightarrow 4X_1 + 6X_2 + 7X_3 = 3$

$\boxed{X_1 = 5.4}$

الاخوة بال matrix
حيث لازم

$[\quad]^{(T)}$ وممكن يكونو

$\begin{array}{c} X_1 \\ X_2 \\ X_3 \end{array}\begin{bmatrix} \\ \\ \\ \end{bmatrix}$

$\Rightarrow$ Singular system

$\hookrightarrow$ لازم اطلع كلمحدد

(Det = 0) ✓

is it singuler?

(I) singwlar system with infint Solutions.

$$\left[\begin{array}{cc:c} 2 & -2 & 4 \\ 4 & -4 & 8 \end{array}\right]$$

a. $-8+8=\underline{0}$

b. gous elemintion

$$\left(\frac{4}{2}\right) R_1 + R_2 \Rightarrow R_2$$

$$\cancel{\begin{bmatrix} 4 & -4 & \vdots & 8 \\ 0 & 0 & \vdots & 0 \end{bmatrix}} \Rightarrow \begin{bmatrix} 2 & -2 & \vdots & 4 \\ 0 & 0 & \vdots & 0 \end{bmatrix}$$

$$0X_2 + X_1 = \underline{\underline{0}} \checkmark$$

لـ أي قيمة لـ x_1, x_2 تعطي (Solution).

(II) Singular system with no solution

$$\begin{array}{l} \text{pivot} \end{array} \begin{bmatrix} 2 & -2 & \vdots & 4 \\ \boxed{4} & -4 & \vdots & 6 \end{bmatrix} \rightarrow D = -8 + 8 = \underline{\underline{0}} \checkmark$$

singular

$$\left(\frac{4}{2}\right) R_1 + R_2 \Rightarrow R_2$$

$$\begin{matrix} 4 & -4 & \vdots & 6 \\ 0 & 0 & \vdots & -2 \end{matrix} \Rightarrow \begin{bmatrix} 2 & -2 & \vdots & 4 \\ 0 & 0 & \vdots & -2 \end{bmatrix}$$

$$\underline{\underline{0}}X_1 + \underline{\underline{0}}X_2 = \underline{\underline{-2}}$$

[Singular (no solution)]